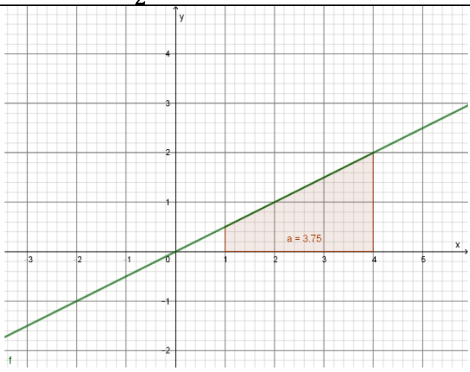
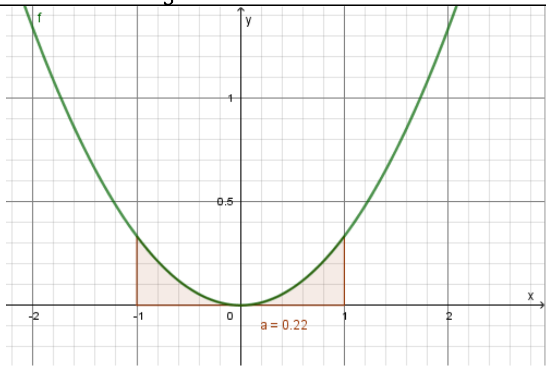
Integralrechnung

Hauptsatz der Differential- und Integralrechnung : $A = \int_a^b f(x)\,dx = [F(x)]_a^b = F(b) - F(a)$

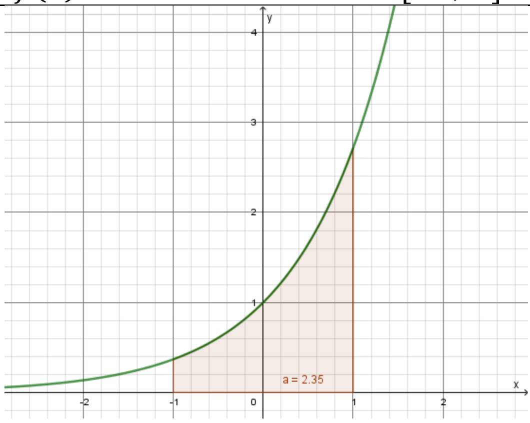
$f(x) = \frac{1}{2}x$ im Intervall I = [1; 4]



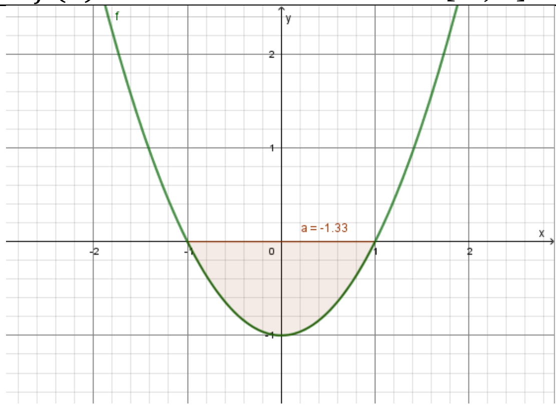
$f(x) = \frac{1}{3}x^2$ im Intervall I = [-1; 1]



$f(x) = e^x$ im Intervall $I = [-1;\ 1]$

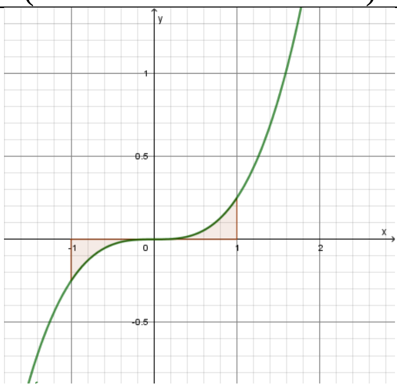


$f(x) = x^2 - 1$ im Intervall I = [-1; 1]

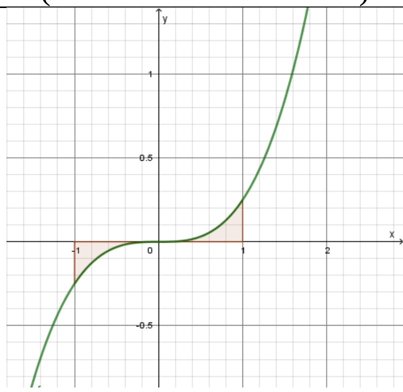


Erkenntnis:

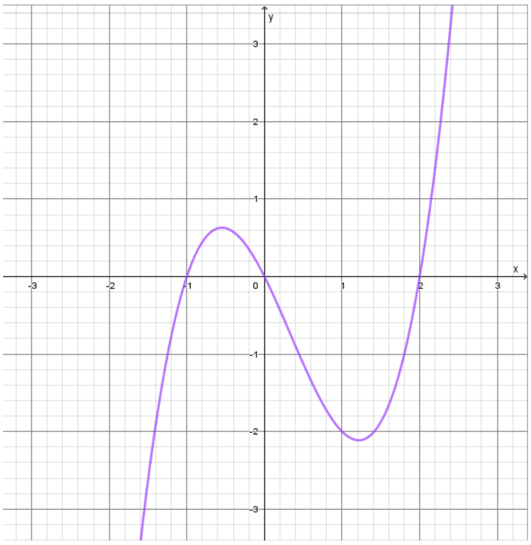
$f(x) = \frac{1}{3}x^3$ im Intervall I = [-1; 1]
(Orientierter Flächeninhalt)



$f(x) = \frac{1}{3}x^3$ im Intervall I = [-1; 1]
(Absoluter Flächeninhalt)



Berechne den Flächeninhalt im Intervall $f(x) = x^3 - x^2 - 2x$ im Intervall I = [-1; 2]



1. Nullstellen berechnen

$x^3 - x^2 - 2x \ = 0$

x (x^2 -x - 2) = 0

1. Fall: x = 0 => $N_1(0|0)$

2. Fall: x^2 -x - 2 = 0

$$x_{1,2} = +\frac{1}{2} \pm \sqrt{\frac{1}{4} + 2}$$

$$x_{1,2} = +\frac{1}{2} \pm \sqrt{\frac{9}{4}}$$

$$x_{1,2} = +\frac{1}{2} \pm \frac{3}{2}$$

$x_1 = +\frac{1}{2} + \frac{3}{2} = \frac{4}{2} = 2$ $N_2(2|0)$ $\quad x_2 = +\frac{1}{2} - \frac{3}{2} = -\frac{2}{2} = -1$ $N_3(-1|0)$

2. Abschnittsweise integrieren

$$A_1 = \int_{-1}^{0} (x^3 - x^2 - 2x)\, dx = \left[\frac{1}{4}x^4 - \frac{1}{3}x^3 - x^2\right]_{-1}^{0} = 0 - \left(\frac{1}{4} + \frac{1}{3} - 1\right) = 0 + \frac{5}{12} = +\frac{5}{12}\ FE$$

$$A_2 = \int_{0}^{2} (x^3 - x^2 - 2x)\, dx = \left[\frac{1}{4}x^4 - \frac{1}{3}x^3 - x^2\right]_{0}^{2} = \frac{1}{4} \cdot 2^4 - \frac{1}{3} \cdot 2^3 - 2^2 - (0) = 4 - \frac{8}{3} - 4 = -\frac{8}{3}$$

$$= -\,2\frac{2}{3}\ FE$$

Absoluter Flächeninhalt: $\frac{5}{12} + \left|-\frac{8}{3}\right| = \frac{5}{12} + \frac{8}{3} = \frac{37}{12} = 3\,\frac{1}{12}\ FE$

Orientierter Flächeninhalt: $\frac{5}{12} - \frac{8}{3} = \frac{5}{12} - \frac{32}{12} = -\frac{27}{12} = -2\,\frac{3}{12} = -2\,\frac{1}{4}\ FE$